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Scalar Absorption by Noncommutative D3-branes

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Abstract

The classical cross section for low energy absorption of the RR-scalar by a stack of noncommutative D3-branes in the large NS B-field limit is calculated. In the spirit of AdS/CFT correspondence, this cross section is related to two point function of a certain operator in noncommutative Yang-Mills theory. Compared at the same gauge coupling, the result agrees with that of obtained from ordinary D3-branes, consistent with the expectation that ordinary and noncommutative Yang-Mills theories are equivalent at long distances.

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Theories on noncommutative spaces arise naturally in string-M theory. In [1], the system of parallel D-branes has been reinterpreted as a quantum space [2]. DLCQ of M-theory with nonzero background three-form field along the null compactified direction has been shown to have matrix theory description of the gauge theory on noncommutative torus [3]. It has been shown in [4] that the D-brane world volume theories are noncommutative at a certain limit of the compactification moduli. Applying Dirac's constrained quantization method to open strings in the presence of NS two-form (B) field, the spacetime coordinates of open string end points have been shown not to commute [5][6]. Recently, by a direct string theoretic analysis, it has been shown in [7] that noncommutativity in the effective action is natural in the presence of constant background B-field.

A particularly interesting example of all these is the noncommutative Yang-Mills theory (NCYM) in four dimensions. The spectrum of IIB theory in the presence of D3-branes on constant B-field consists of open and closed string excitations coupled together. However, it is possible to take a low energy limit and scale some parameters such that the closed string modes decouple [7]. The resulting theory of open string modes turn out to be the NCYM. In [8][9], a supergravity background has been proposed to be dual to this system. This background can be obtained by first constructing a solution which has nontrivial B-field dependence and then taking the decoupling limit of [7]. Closed string two point scattering amplitudes in the presence of B-field have been calculated and shown to be consistent with the ones encoded in the gravity solution [11]. In [10], the solution has been shown to have holographic features. Some properties of the NCYM have also been studied using its gravity dual [9][12][13].

In this paper we will consider the process of classical low energy absorption of the RR-scalar by noncommutative D3-branes in the large B-field limit. In the spirit of AdS/CFT correspondence, the cross section calculated from the gravity side is related to discontinuity of two point function of a certain operator in the dual NCYM [20]. This operator can be deduced from the coupling of the scalar to the D-brane world volume effective action [14]. The fluctuations turn out to be non-minimally coupled to background geometry. However, the coupling of RR-scalar to the effective world volume theory is relatively simple which may help one to identify the operator in NCYM. In calculating the absorption cross section, we will work with the D3-brane solution of [9] before taking the decoupling limit. As discussed in the context of ordinary D3-branes in [21], the decoupling limit identifies the so called throat region with a certain limit [2] of the world volume theory. On the other hand, a large B-field is also encountered in the decoupling limit considered in [7] [3], which means that the dual theory is "close" to its throat limit. For previous work on cross section calculations on black-brane backgrounds see, for instance, [15]-[22].

The type IIB supergravity action in the string frame can be written as

$$S_{IIB} = \frac{1}{2\kappa_{10}^2} \int dx^{10} \sqrt{-g} (e^{-2\phi}[R + 4(\partial\phi)^2 - \frac{3}{4}(\partial B_2)^2]$$
$$-\frac{1}{2}(\partial C)^2 - \frac{3}{4}(\partial C_2 - C\partial B_2)^2 - \frac{5}{6}F(C_4)^2 - \frac{\epsilon_{10}}{48} C_4 \partial C_2 \partial B_2 + ...) \tag{1}$$

where $\partial B_2 = \partial_{[\mu} B_{\nu\lambda]}$ etc., $F(C_4) = \partial C_4 + 3/4(B_2 \partial C_2 - C_2 \partial B_2)$, and the self duality of $F(C_4)$ is imposed at the level of field equations. Specifically B_2 denotes the NS two-form field and C is the RR-scalar. The solution corresponding to noncommutative D3-branes is given by [9],

$$ds^2 = f^{-1/2}[-dt^2 + dx_1^2 + h(dx_3^2 + dx_4^2)] + f^{1/2}[dr^2 + r^2 d\Omega_5^2], \tag{2}$$

$$B_{23} = \frac{\sin\theta}{\cos\theta} f^{-1} h, \quad e^{2\phi} = g_s^2 h, \quad \partial(C_2)_{01r} = \frac{1}{g_s} \sin\theta \partial_r f^{-1}, \tag{3}$$

$$F(C_4)_{0123r} = \frac{1}{g_s} \cos\theta h \partial f^{-1}, \tag{4}$$

where g_s is the asymptotic value of the string coupling constant, and the functions f and h are given by

$$f = 1 + \frac{R^4}{r^4}, \qquad h = \frac{f}{\sin^2\theta + \cos^2\theta f}. \tag{5}$$

The solution is asymptotically flat and there is a horizon at $r = 0$. The near horizon geometry is $AdS_5 \times S_5$. The mass per unit volume of (2) in string frame can be calculated to be

$$M = \frac{2\pi^3 R^4}{\kappa_{10}^2 g_s^2}, \tag{6}$$

which is remarkably independent of θ. Note that the asymptotic value of B-field is $\tan\theta$.

[2] The conformal limit in the case of ordinary D3-branes.

[3] In taking the decoupling limit one keeps B fixed and let α' go to zero which corresponds to a very large B-field for finite (but small) α'.

From (1), the fluctuations of the RR-scalar on this background obeys

$$\nabla^2 C = \frac{3}{2}(\partial B_2)^2 C, \tag{7}$$

and thus couple to the background geometry non-minimally. We note that the contraction $(\partial B_2)(\partial C_2)$ is zero. We will assume that C does not depend on the spatial coordinates of D3-branes. Separating the time dependence and considering a spherically symmetric fluctuation (s-wave), which is supposed to give the dominant contribution to cross section, we write

$$C = e^{-iwt}\phi(r). \tag{8}$$

Following from (7), ϕ obeys

$$(hr^5)^{-1}\frac{d}{dr}(hr^5\frac{d}{dr}\phi) + \omega^2 f\phi - \frac{16\sin^2\theta\cos^2\theta R^8}{r^{10}} f^{-3}h^2\phi = 0. \tag{9}$$

Since this equation does not appear to be analytically solvable, following previous work, we try to find an approximate solution by matching three different regions dictated by the structure of the functions f and h. Low energy scattering is characterized by $\omega\sqrt{\alpha'} \ll 1$ and the α' corrections to background is suppressed when $\sqrt{\alpha'}/R \ll 1$. Consistent with these two restrictions, we will consider the double scaling limit of [18] and assume $\omega R \ll 1$. Large B-field corresponds to $\cos\theta \ll 1$ and we will further analyze the case where $\cos\theta \sim \omega R$.

Region I: $r \gg R$

In this region $f \sim h \sim 1$. Defining $\rho = \omega r$ and $\phi = \rho^{-5/2}\psi$, (9) simplifies as

$$\left(\frac{d^2}{d\rho^2} - \frac{15}{4\rho^2} + 1 - \frac{16\sin^2\theta\cos^2\theta(\omega R)^8}{\rho^{10}}\right)\psi = 0. \tag{10}$$

Since $\rho \gg \omega R$, the last term in this equation is negligible compared to the second one. Ignoring this term, (10) can be solved in terms of Bessel and Neumann functions which in turn gives

$$\phi = a_1\ \rho^{-2}J_2(\rho) + a_2\ \rho^{-2}N_2(\rho), \tag{11}$$

where a_1 and a_2 are constants.

Region II: $R \gg r \gg R\sqrt{\cos\theta}$

In this region f and h can be approximated as $f \sim h \sim R^4/r^4$. Using this form and defining $\phi = \rho^{-1/2}\chi$, (9) becomes

$$\left(\frac{d^2}{d\rho^2} + \frac{1}{4\rho^2} + \frac{(\omega R)^4}{\rho^4} - \frac{16\sin^2\theta\cos^2\theta(\omega R)^4}{\rho^6}\right)\chi = 0. \tag{12}$$

In region II, $\omega R \gg \rho \gg \omega R\sqrt{\cos\theta}$ and in that interval; the third term can be ignored compared to the fourth one (with the assumption $\cos\theta \sim \omega R$), and the fourth term is always very small with respect to the second one. Therefore, χ approximately obeys

$$\frac{d^2}{d\rho^2}\chi + \frac{1}{4\rho^2}\chi = 0. \tag{13}$$

Two solutions of this equation are $\rho^{1/2}$ and $\rho^{1/2}\ln\rho$, and thus

$$\phi = b_1 + b_2\ \ln\rho, \tag{14}$$

where b_1 and b_2 are constants.

Region III: $R\sqrt{\cos\theta} \gg r$

In this region $f \sim R^4/r^4$ and $h \sim 1/\cos^2\theta$. Defining $z = \omega R\sqrt{\cos\theta}/\rho$ and $\phi = z^{3/2}Z$, equation (9) can be approximated to,

$$\left(\frac{d^2}{dz^2} - \frac{15}{4z^2} + \frac{(\omega R)^2}{\cos\theta} - 16\frac{\sin^2\theta}{z^6}\right) Z = 0. \tag{15}$$

In region III, $z \gg 1$ and thus the last term can be ignored compared to the second one. Dropping this term, two solutions of Z can be found to be $z^{1/2}J_2(z\omega R/\sqrt{\cos\theta})$ and $z^{1/2}N_2(z\omega R/\sqrt{\cos\theta})$. This gives

$$\phi = c_1\ \rho^{-2}J_2\left(\frac{(\omega R)^2}{\rho}\right) + c_2\ \rho^{-2}N_2\left(\frac{(\omega R)^2}{\rho}\right), \tag{16}$$

where c_1 and c_2 are constants.

Matching the solutions:

In matching the solutions in different regions, we will use the small argument expansion of the Bessel and Neumann functions

$$\begin{aligned} J_2(x) &\sim \frac{x^2}{8}, \\ N_2(x) &\sim \frac{-4}{\pi x^2}(1+\frac{x^2}{4}) + \frac{1}{4\pi}x^2(\ln x + c), \end{aligned} \tag{17}$$

where c is a constant. Close to the horizon, we want only an ingoing wave which implies

$$c_1 = -i\,c_2. \tag{18}$$

The overall normalization of ϕ can be fixed by imposing $c_1 = i(\omega R)^4$. To be able to match the solution (16) to region II, we consider its behavior when $\rho \sim \omega R\sqrt{\cos\theta}$.

Assuming $\cos\theta \sim \omega R$, the arguments of the Bessel and Neumann functions in (16) are small in this range and thus we can use the expansion (17). We see that there is no term in this expansion to be matched by $\ln\rho$ of region II, therefore, we should set $b_2 = 0$. The dominant contribution of the rest of the terms when $\rho \sim \omega R\sqrt{\cos\theta}$ is the constant $4/\pi$, which fixes b_1 as

$$b_1 = \frac{4}{\pi}. \tag{19}$$

To match (11) to region II, we will consider its behavior when $\rho \sim \omega R$. The arguments of Bessel and Neumann functions in (11) are also small in this range. The leading contributions of their expansions are $a_1/8$ and $(-4a_2)/(\pi\rho^4)$, respectively. To be able to match these to (14), one should set

$$a_2 = 0, \qquad a_1 = \frac{32}{\pi}. \tag{20}$$

Combining these, we obtain the following functions in three regions

$$\begin{aligned} \phi_I &= \frac{32}{\pi}\rho^{-2}J_2(\rho), \\ \phi_{II} &= \frac{4}{\pi}, \\ \phi_{III} &= i(\omega R)^4\rho^{-2}\left[J_2\left(\frac{(\omega R)^2}{\rho}\right) + iN_2\left(\frac{(\omega R)^2}{\rho}\right)\right], \end{aligned} \tag{21}$$

which smoothly overlaps and give an approximate solution to (9)[4]. To calculate the cross section one should compare the incoming flux at the horizon with the incoming flux at the infinity. At this stage, we recognize that the same functions appear in [18] in the solutions of the massless wave equation on ordinary D3-brane background. The cross section corresponding to the solution (21) can be read from [18] to be

$$\sigma_{abs} = \frac{\pi^4}{8}\omega^3 R^8. \tag{22}$$

We now try to rewrite σ_{abs} in terms of the gauge coupling constant $\tilde{g}_{YM}$ of NCYM.

The solution (2) can be shown to preserve 1/2 supersymmetries of the theory. This can easily be seen by nothing that (2) is related to ordinary D3-brane solution by a chain of T-duality transformations and T-duality respects supersymmetry when the Killing spinor is independent of the direction of the duality [23]. From a world-sheet

[4]To ensure that ϕ can be differentiated twice, one has to let $b_2 = O(\omega R)$ and $a_2 = O((\omega R)^5)$, instead of setting them to zero. To zeroth order in ωR, this modification does not change the main result (22).

point of view, one can also see that the parallel D3-branes on constant, invertible, B-field backgrounds also preserve 1/2 supersymmetries since boundary conditions identify the left moving supercurrents with the right moving ones. This is consistent with the identification of this configuration with the gravity solution. Due to this BPS property, the noncommutative D3-brane tension, when calculated from an effective action point of view, should be equal to the mass per unit volume (6). We will now carry out this effective field theory calculation to fix the value of R.

The Dirac-Born-Infeld action corresponding to a noncommutative D3-brane can be written as

$$S_{DBI} = T_3 \int d^4\sigma \sqrt{-\det(\hat{g} + \hat{B})}, \tag{23}$$

where σ^i are coordinates on the D3 brane, T_3 is the ordinary D3-brane tension when $B = 0$, $\hat{g}$ and $\hat{B}$ are pull-backs of flat Minkowski metric $\eta_{\mu\nu}$ and constant B-field $B_{\mu\nu}$, respectively,

$$\begin{aligned} \hat{g}_{ij} &= \partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu}, \qquad (24) \\ \hat{B}_{ij} &= \partial_i X^\mu \partial_j X^\nu B_{\mu\nu}. \qquad (25) \end{aligned}$$

From S_{DBI}, one can calculate the conjugate momentum densities to coordinate fields X^μ as

$$P_\mu = \frac{\delta S_{DBI}}{\delta \partial_\tau X^\mu}, \tag{26}$$

where τ is the world volume time coordinate.

The tension $\tilde{T}_3$ of a noncommutative D3-brane can be defined as the energy density corresponding to a flat, non-exited brane. This can be described in a physical gauge by $X^i = \sigma^i$, $X^\alpha = const$. For such a brane, (26) gives

$$P_\mu = T_3 \sqrt{-\det(\eta_{ij} + B_{ij})}\ \delta^0_\mu, \tag{27}$$

which corresponds to the momentum density of a massive object with zero velocity. The energy per unit volume of such an object can be read from P_0 component of the momentum. This gives the tension for a noncommutative D3-brane as

$$\tilde{T}_3 = T_3 \sqrt{-\det(\eta_{ij} + B_{ij})}. \tag{28}$$

Note that the ordinary D3-brane tension and the 10-dimensional gravitational coupling constant are

$$T_3 = \frac{1}{(2\pi)^3 \alpha'^2 g_s}, \qquad 2\kappa_{10}^2 = (2\pi)^7 \alpha'^4. \tag{29}$$

In the solution (2), B-field is a rank 2 matrix. Using (28) and (29) we obtain,

$$\tilde{T}_3 = \frac{1}{(2\pi)^3 \alpha'^2 g_s \cos\theta}. \tag{30}$$

The total energy of N-coincident D3-branes is given by $N\tilde{T}_3$ which should be equal to (6). This gives the parameter R as[5]

$$R^4 = \frac{4\pi\alpha'^2 g_s N}{\cos\theta}. \tag{31}$$

On the other hand the gauge coupling constant $\tilde{g}_{YM}$ of NCYM, can be read from [7] to be

$$\begin{aligned} \tilde{g}_{YM}^2 &= 2\pi g_s \sqrt{-\det(\eta_{ij} + B_{ij})}, \\ &= \frac{2\pi g_s}{\cos\theta}. \end{aligned} \tag{32}$$

Note that for $\theta = 0$, this gives the well known relation between the ordinary Yang-Mills and string coupling constants. Combining (32) with (31), the cross section (22) can be rewritten in terms of $\tilde{g}_{YM}$ as

$$\sigma_{abs} = \frac{\pi^4}{2}\omega^3 \alpha'^4 N^2 \tilde{g}_{YM}^4. \tag{33}$$

Remarkably, all θ dependence is hidden in $\tilde{g}_{YM}$. The classical cross section for low energy absorption of RR-scalar by ordinary D3-branes (which is identical to massless scalar absorption) has been calculated in [18] and the result is (33) in which $\tilde{g}_{YM}$ is replaced with the gauge coupling g_{YM} of ordinary Yang-Mills theory. Comparing NCYM with the ordinary Yang-Mills at the same coupling

$$\tilde{g}_{YM} = g_{YM}, \tag{34}$$

the result of [18] exactly agrees with (33). This is consistent with the expectation that ordinary and noncommutative Yang-Mills theories are equivalent at long distances.

As previously noted, these cross sections are related to two point functions of certain operators in dual theories. This operator can be deduced from the coupling of

[5] Exactly the same expression for R is given in [9], which is obtained by T-duality from the ordinary D3-brane solution.

the scalar at hand to the effective world volume theories. For the ordinary Yang-Mills theory, leading order coupling of RR-scalar to the world volume is

$$\epsilon^{ijkl} C \text{ Tr } F_{ij} \ F_{kl}. \tag{35}$$

The classical cross section for the absorption of RR-scalar by ordinary D3-branes has been shown to agree with a tree level world volume calculation which involves the above coupling [19]. This indicates that the leading term of corresponding operator in the dual theory is ϵ^{ijkl} Tr F_{ij} F_{kl}.

Naturally, one may try to identify the coupling of RR-scalar to a noncommutative D3-brane world volume. As discussed in [7], it is very convenient to write the effective action in terms of the noncommutative gauge fields and open string parameters. When expressed in these variables, the effective action in the presence of B-field can be deduced form the ordinary one. From (35), the coupling of RR-scalar to the noncommutative D3-brane world volume can be written as

$$\epsilon^{ijkl} C * \text{Tr} \hat{F}_{ij} * \hat{F}_{kl}, \tag{36}$$

where $\hat{F}$ is the noncommutative gauge field strength, ϵ-tensor and raising of indices refer to open string metric and $*$-product is defined by

$$(f * g)(x) = e^{\frac{1}{2}\theta^{ij} \frac{\partial}{\chi^i} \frac{\partial}{\eta^j}} f(x+\chi) g(x+\eta)|_{\chi=\eta=0}. \tag{37}$$

The open string metric and noncommutativity parameter θ^{ij} is fixed in terms of the background Minkowski metric and B-field [7]. This indicates that the leading order term of the operator in the NCYM is ϵ^{ijkl} Tr $\hat{F}_{ij} * \hat{F}_{kl}$. Note, however, that this term is not gauge invariant but it is gauge covariant.

One may try to repeat same calculations for noncommutative M5-branes using the solution given in [9]. For ordinary M5-branes, the traceless metric perturbations polarized along the 5-branes have been shown to obey minimally coupled scalar equations [19]. One can easily show that this is also true for noncommutative M5-branes. Furthermore, the massless scalar equation on noncommutative M5-branes turns out to be the same with the one on the ordinary M5 branes. Therefore, the classical cross sections corresponding to absorption of traceless metric perturbations polarized along the 5-branes are identical for both type of branes.

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